NASA CONTRACTOR
REPORT

N71-35436

NASA CR-61359

ATMOSPHERIC DENSITY VARIATIONS RELATED TO INTERNAL GRAVITY WAVES

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Huntsville, Alabama

August 1971

CASE FILE COPY

Prepared for

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama 35812

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO. NASA CR-61359	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
4. TITLE AND SUBTITLE ATMOSPHERIC DENSITY VARIATIONS RELATED TO INTERNAL GRAVITY WAVES		5. REPORT DATE August 1971 6. PERFORMING ORGANIZATION CODE
7. AUTHOR (S) E. B. Miller		8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND ADDRESS Northrop Corporation Huntsville, Alabama		10. WORK UNIT NO. 11. CONTRACT OR GRANT NO. NAS8-20082
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D. C. 20546		13. TYPE OF REPORT & PERIOD COVERED Contractor Report 14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

Technical Coordinator: Robert E. Smith, Aerospace Environment Division, Aero-Astrodynamics Laboratory

16. ABSTRACT

A method for estimating gravity-wave density variation from wind speed profiles between 60 km and 100 km is outlined. The effects of background wind shear and mean temperature appear analytically. Under certain approximations, the background wind shear is seen to have no effect upon the magnitude of gravity wave density variations, although the mean temperature and its vertical gradient may be more significant in this respect. Consideration is given to determining background or mean values from typical vertical profiles of either wind speed or temperature. It is concluded that a single profile, of the usual sort, is inadequate for a valid determination of these mean or background values. Ensemble averaging over several related profiles is recommended. An estimate of density variations is made from the ensemble averaging of 70 mid-latitude wind profiles which was carried out by Rosenberg (1968). This estimate shows density variations between 85 km and 100 km to be about 20 percent of the background density.

17. KEY WORDS gravity-wave density variation, wind speed profiles, ensemble averaging	18. DISTRIBUTION STATEMENT Unclassified-Unlimited E. D. GEISSLER, Director, Aero-Astrodynamics Laboratory, MSFC

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	40	$3.00

MSFC - Form 3292 (May 1969)

FOREWORD

A method is outlined in this document for estimating density variations in the atmosphere. Wind data which have the characteristics of internal gravity waves are basic to this method. A brief review of the gravity wave theory is also presented. The work was conducted by Northrop Corporation, Huntsville, Alabama, for the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, Aero-Astrodynamics Laboratory, under Contract NAS8-20082, Appendix A-1, Schedule Order 32. This work was under the direction of the Space Environment Branch, with Mr. R. E. Smith as NASA/ MSFC Technical Coordinator.

ACKNOWLEDGEMENTS

The author wishes to thank Richard L. King for many helpful discussions. Thanks are also due to H. Prescott Sleeper for his review of the literature pertaining to upper atmospheric data and measurement. The author is especial grateful to Prof. C. O. Hines for detecting an inconsistency in an early draft of this document.

TABLE OF CONTENTS

LIST OF SYMBOLS

Roman

Symbol	Definition
c	the speed of sound
C_v	specific heat per unit mass (at constant volume)
D/Dt	"substantial" or "material" derivative
E_k	gravity-wave kinetic energy
f	factor defined here by equation (10)
g	the acceleration of gravity
G	a convenient grouping of parameters defined in equation (10)
k	the thermal conductivity
k_z	the real part of K_z
K_n	the Knudsen number, ℓ/λ_z
K_x, K_z	wave numbers in the x and z directions, respectively, K_z complex
ℓ	the mean free path
ℓ_z	the imaginary part of K_z
p'	a pressure fluctuation
P	pressure
P	a proportionality constant in equation (26)
P_o	a ground-state pressure
Q	a proportionality constant in equation (26)
R	the specific gas constant
Ri	the Richardson number, equation (46)
t	time
T_o	the ground state temperature

LIST OF SYMBOLS (Concluded)

Symbol	Definition
U_o	the background wind speed in the x-direction
V_o	the background wind speed in the y-direction
x	a coordinate used in describing horizontal motion
X	a proportionality constant in equation (26)
y	a coordinate used in describing horizontal motion
z	a coordinate used in describing vertical motion
z_o	a reference altitude
Z	a proportionality constant in equation (26)
u', v', w'	velocity fluctuations in the x, y, and z directions, respectively

Greek

Symbol	Definition
γ	specific heat ratio
λ_x, λ_z	typical horizontal and vertical wavelengths, respectively
ν	the kinematic viscosity
ρ	density
ρ_o	the ground state-density
ρ'	a density fluctuation
τ	a typical period
ϕ	a parameter defined by equation (31)
ω	angular frequency in a system fixed to the earth's surface
ω_g	the Brunt-Väisälä frequency
ω_a	the acoustic cut-off frequency
Ω	the frequency in a frame of reference moving at the wind speed

Section I

INTRODUCTION

A significant portion (10^3 seconds) of the total reentry time for the space shuttle orbiter will be spent in traversing the altitude range between 60 km and 100 km. Densities at these altitudes are therefore of interest from the standpoints of trajectory, control, and heating.

Static atmospheric models are known to be inadequate for predicting satellite impact and the decay of satellite orbits (e.g., see Schilling (1968))*. DeVries (1971) notes that these models assume static equilibrium conditions which are inadequate for dealing with short-period atmospheric perturbations. Thus, these static models are certainly inadequate for estimating density variations which occur with periods on the order of 10^3 seconds.

Density variations may be associated with a number of physical phenomena including turbulence, auroral activity, atmospheric chemistry, atmospheric heating, and "gravity waves". Wind-speed profiles are relatively plentiful between 60 km and 100 km. These may be obtained by a number of methods, including the radar tracking of chaff, optical tracking of the chemiluminescent trails resulting from rocket releases, acoustical techniques, and the radar tracking of the ionization trails of meteors. Many of these data show evidence of "gravity waves" or waves resulting from the effects of compressibility and bouyancy.

Hines (1960) was the first to relate the salient features of meteor-trail wind data (available between 80 km and 115 km) to internal gravity waves. In summarizing features of these data, he notes that they are characterized by roughly horizontal winds which exhibit strong variations over vertical distances of a few kilometers. A more or less typical variation is seen to have: a vertical scale of about 12 km; a horizontal scale which exceeds the vertical

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A complete reference on each author's work cited in this report is given in Section VI.

scale by a factor of 20 or more; and a period of about 200 minutes. The wind variations are generally seen to increase with height and a background wind shear is frequently evident.

This report is concerned with obtaining an analytic expression which permits the use of gravity wave wind-profiles in making estimates of the magnitude of density variations between the altitudes of 60 km and 100 km. A review of the gravity wave theory is also included.

Hines (1960) has made the primary contribution in this direction by developing an analytical expression relating density variations to wind speed variations for gravity waves. It is not clear, however, to what extent his analytic expression is valid for an atmosphere without a constant mean temperature or having a background wind.

Pitteway and Hines (1965) have obtained analytic expressions for the linear and exponential variation of the mean temperature with height by employing a W. K. B. approximation. However, they did not obtain an analytic expression which is suitable for the estimation of density variations from wind variations in the presence of background wind shear.

Hines and Reddy (1967) have considered the effect of wind speed in their analysis of the propagation of atmospheric gravity waves through regions of wind shear. However, they have neglected an explicit background wind-shear parameter in their formulation of the problem. The background wind shear is introduced by dividing the atmosphere into layers, each possessing a constant wind speed. As noted by Booker and Bretherton (1967) and by Hines (1968) this approach leads to problems at "critical layers" where the background wind speed in the direction of horizontal wave propagation equals the horizontal phase speed. Since this analysis fails to deal analytically with background wind-shear, it cannot be adapted to the analytical estimation of density variations when wind shear is present.

Bretherton (1966) and Booker and Bretherton (1967) have included the effects of wind shear in their analyses. However, these analyses are based on the Bousinnesq equations which are only valid for gravity waves in a compressible fluid when the vertical scale of the motion is small compared to the atmospheric scale height (e.g., see Bretherton (1966), Dutton and Fichtl (1969)). For the altitude range of interest here, the vertical scale of motion may be on the same order as the scale height so that the results of these workers may not be generally applicable to the problem treated in this report.

This report attempts to introduce wind shear and temperature gradients into the gravity wave problem while retaining the simple formulation of Hines (1960). This will be seen to impose certain restrictions upon the generality of the results. These restrictions are considered in some detail and they are not found to seriously inhibit the estimation of density variations from wind data.

The report is organized into five sections. Section II is concerned with the basic theory. Attention is also given to the justification of the assumptions which are used in establishing and specializing the basic equations. Section III develops and justifies the expression for estimating the variations in density associated with velocity variations in the presence of wind shear and temperature gradients. Attention is also given to critical levels, temperature and wind profiles and stability. Section IV discusses the physical significance of parameters and parameteric regimes. The use of the results of Section III is also demonstrated for a particular sample of data. Section V summarizes the principal results and conclusions. Section VI contains a list of references to the work of each author cited in this report. An appendix is also included which discusses the problem of estimating the background wind speed and the mean temperature from available data.

Section II

THEORY

In this section the Navier-Stokes equations will be specialized to give a simple analytical relation between density and velocity variations assiciated with low-frequency gravity waves, for altitudes between 60 km and 100 km. However, before proceeding with this, the constraints imposed by the continuum assumption will be investigated. This assumption is basic to the Navier-Stokes equations.

Kinetic theory may be used to show that the continuum gas dynamics regime exists provided that the Knudsen number, K_n, satisfies the condition

$$K_n \leq 10^{-2} \tag{1}$$

The Knudsen number for internal gravity waves is defined as the ratio of the mean free path to the dominant vertical scale, or vertical wavelength, of the motion. That is

$$K_n = \frac{\ell}{\lambda_z} \tag{2}$$

The mean free path, ℓ, as given by the U. S. Standard Atmosphere (1962) is on the order of 10^{-4}m at 60 km and 10^{-1}m at 100 km. Thus, from condition (1), it follows that the Navier-Stokes equations may be applied between 60 km and 100 km provided that they are used to describe vertical wavelengths greater than about 1 cm at 60 km and greater than about 10 m at 100 km. These limiting values are small in comparison with the nominal vertical scale for internal gravity waves of about 10^4m, which Hines suggests for altitudes between 80 km and 115 km. Thus, the Navier-Stokes equations may be safely applied.

The important conclusion which results from condition (1) and equation (2) is that the usefulness of the Navier-Stokes equations is limited by the vertical wavelength of the motion and the molecular mean free path of the atmosphere. Since the mean free path increases upward, then so must the limiting vertical wavelength associated with the motion. Physically, condition (1) says that the wavelength of the motion must at least be of a certain magnitude or the ordered wave motion will be lost in the chaos of random molecular interaction. Waves having a specified vertical wavelength may be considered to propagate upward until condition (1) is violated. Above this limiting point it should be impossible to detect ordered wave motion of the specified wavelength.

2.1 SPECIALIZATION OF THE BASIC EQUATIONS

In analyzing gravity wave phenomena between 60 km and 100 km, chemistry, hydromagnetic effects, radiation heat transfer, and atmospheric turbulence will be neglected, as is usual. Rotation or Coriolis effects and earth curvature will also be neglected. Of these latter two effects, the neglect of rotation appears to impose the most serious restriction. Tolstoy (1967) indicates that a rotation-free formulation is only useful in the description of wave phenomena having periods less than about 3 or 4 hours.

After the neglect of these effects, the basic equations are:

$$\frac{D\vec{V}}{Dt} = -\frac{\vec{\nabla} P}{\rho} - g\hat{k} + \nu\vec{\nabla}^2\vec{V} + \frac{\nu}{3}\vec{\nabla}(\vec{\nabla}\cdot\vec{V}) \tag{3}$$

(the momentum equations)

$$\frac{D\ell n\rho}{Dt} = -\vec{\nabla}\cdot\vec{V} \tag{4}$$

(the continuity equation)

$$\frac{DT}{Dt} + \left(\frac{P}{\rho C_v}\right)\vec{\nabla}\cdot\vec{V} = \left(\frac{k}{\rho C_v}\right)\vec{\nabla}^2 T + \frac{\nu}{C_v}\vec{\nabla}\cdot(\vec{V}\cdot\vec{\nabla}\vec{V}) - \frac{2}{3}\frac{\nu}{C_v}(\vec{\nabla}\cdot\vec{V})^2 \tag{5}$$

(the energy equation)

$$P = \rho RT \tag{6}$$

(the state equation)

These equations are in terms of the vector velocity, $\vec{V}$, the pressure, P, the density, ρ, the temperature, T, the kinematic viscosity, ν, the specific heat per unit mass (at constant volume), C_v, the specific gas constant of air, R, and the thermal conductivity, k. The notation D/Dt denotes the "substantial" or "material" derivative. This operator is defined by

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \qquad (7)$$

Under certain conditions viscosity and thermal conductivity can be important. The neglect of these effects should be carefully considered. A scale analysis may be used to estimate the relative orders of magnitude of viscous and thermal conduction terms in the basic equations.

Two relations which are helpful in a scale analysis of equations (3) and (5) are

$$\gamma \equiv 1 + \frac{R}{C_v} \qquad (8)$$

which is obtained from elementary thermodynamic considerations and

$$k = f\ C_v\ \rho\ \nu \qquad (9)$$

from kinetic theory. The factor f in equation (9) may be given by

$$f = \frac{9}{4}\ (\gamma - 5/9) \qquad (10)$$

for non-polar gases of moderate complexity such as O_2, N_2, CO_2, and CH_4 (e.g., see O'Neal and Brokaw (1962)).

Equations (3) and (5) may be non-dimensionalized with respect to a typical period, τ, a typical horizontal wavelength, λ_x, and a typical vertical wavelength, λ_z. Based on the characteristics of wind data at the stated altitudes, it will also be presumed that $\lambda_z < \lambda_x$. For this case comparison of the magnitudes of terms in equations (3) through (5) clearly shows that viscous and thermal conduction effects first become significant when it is no longer possible to state that

$$\frac{\nu\tau}{\lambda_z^2} << 1 \tag{11}$$

Hines (1960) gives 10^4 seconds as the period of a typical gravity wave between 80 km and 115 km. The kinematic viscosity, ν, varies from 10^{-2} m^2/sec to 1 m^2/sec between 60 km and 90 km. Thus between 80 and 90 km, condition (11) gives

$$\lambda_z^2 >> 10^4 \ m^2 \tag{12}$$

However, Hines indicates that $\lambda_z^2 \sim 10^8 \ m^2$ so that inequality (12) is satisfied and both viscosity and thermal conductivity may be neglected up to 90 km, at least.

Scale analysis also demonstrates the importance of the grouping $\lambda_x/U_o\tau$ where U_o is the background wind speed in the x, or horizontal, direction. This grouping, sometimes called the "Strouhal number", indicates the relative importance of the time- versus the space-derivative in the operator defined by equation (7). In subsequent work the Strouhal number will generally appear in the form ω/K_xU_o where K_x is the horizontal wave number, $|K_x| \equiv 2\pi/\lambda_x$, and the angular frequency ω is given by $\omega \equiv 2\pi/\tau$.

2.2 LINEARIZATION OF THE BASIC EQUATIONS

After neglect of terms dependent on viscosity and thermal conductivity, equations (3) through (6) may be linearized to produce a set of perturbation equations and a ground state equation. The thermodynamic parameters, P, ρ, and T are assumed to have the following dependence upon x, y, z and t:

$$P \equiv P_o(z) + p'(x, y, z, t) \tag{13}$$

$$\rho \equiv \rho_o(z) + \rho'(x, y, z, t) \tag{14}$$

$$T \equiv T_o(z) + T'(x, y, z, t) \tag{15}$$

The velocity components are similarly defined

$$u \equiv U_o(z) + u'(x, y, z, t) \tag{16}$$

$$v \equiv V_o(z) + v'(x, y, z, t) \tag{17}$$

$$w \equiv w'(x, y, z, t) \tag{18}$$

The coordinate system will be chosen so that the x and y axes describe horizontal wave propagation, and the positive z direction corresponds to the direction of waves which propagate vertically upward. Further, the coordinate system will be oriented so that $\partial/\partial y \equiv 0$.

The perturbation equations are obtained from the inviscid and isentropic forms of equations (3) through (6). They are derived by the neglect of terms containing the products of primed quantities. These equations are:

$$\frac{\partial u'}{\partial t} + U_o \frac{\partial u'}{\partial x} + \frac{dU_o}{dz} w' + \frac{c^2}{\gamma} \frac{\partial}{\partial x}\left(\frac{p'}{P_o}\right) = 0 \tag{19}$$

(x-momentum equation)

$$\frac{\partial v'}{\partial t} + U_o \frac{\partial v'}{\partial x} + \frac{dV_o}{dz} w' = 0 \tag{20}$$

(y-momentum equation)

$$\frac{\partial w'}{\partial t} + U_o \frac{\partial w'}{\partial x} + g\left(\frac{\rho'}{\rho_o}\right) + \frac{c^2}{\gamma} \frac{\partial}{\partial z}\left(\frac{\rho'}{\rho_o}\right) + \frac{c^2}{\gamma}\left(\frac{d\ell n\rho_o}{dz}\right)\left(\frac{\rho'}{\rho_o}\right) = 0 \tag{21}$$

(z-momentum equation)

$$\frac{\partial}{\partial t}\left(\frac{\rho'}{\rho_o}\right) + U_o \frac{\partial}{\partial x}\left(\frac{\rho'}{\rho_o}\right) + \frac{d\ell n\rho_o}{dz} w' + \frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \tag{22}$$

(continuity equation)

$$\frac{\partial}{\partial t}\left(\frac{p'}{P_o}\right) + U_o \frac{\partial}{\partial x}\left(\frac{p'}{P_o}\right) + \frac{d\ell n P_o}{dz} w' + \gamma \frac{\partial u'}{\partial x} + \gamma \frac{\partial w'}{\partial z} = 0 \tag{23}$$

(energy equation)

The parameter c^2 is defined by

$$c^2 \equiv \frac{\gamma P_o}{\rho_o} = \gamma R T_o \tag{24}$$

The ground state equation associated with these perturbation equations is

$$\frac{dP_o}{dz} = - \rho_o g \tag{25}$$

Equations (19) through (23) comprise a set of five simultaneous equations in the five variables u', v', w', p'/P_o, and ρ'/ρ_o. The variable, v', is only involved in equation (20); therefore, this equation may be dropped, leaving a set of four equations. The component of background wind, V_o, which is directed across the wavefront, only occurs in the omitted equation so that it cannot affect wave propagation characteristics.

Following Hines (1960), a solution is now sought to the four remaining perturbation equations of the form,

$$\frac{\rho'}{\rho_o Q} = \frac{p'}{P_o P} = \frac{u'}{X} = \frac{w'}{Z} = \exp[i(\omega t - K_x x - K_z z)] \tag{26}$$

where K_x and ω are presumed to be positive, real constants and K_z is a complex constant. The parameters Q, P, X and Z are proportionality constants.

Substitution of equations (26) into equations (19), (21), (22), and (23) results in the following matrix equation

$$\begin{pmatrix} \Omega & -i\, dU_o/dz & -K_x c^2/\gamma & 0 \\ 0 & \Omega & -Gc^2/\gamma & -ig \\ -K_x\gamma & -\gamma\left[G + \frac{ig(\gamma-1)}{c^2}\right] & \Omega & 0 \\ -K_x & -\left(G - i\,\frac{d\ell nT_o}{dz}\right) & 0 & \Omega \end{pmatrix}\begin{pmatrix} X \\ Z \\ P \\ Q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (27)$$

The parameter Ω which appears in system (27) is the angular frequency of the wave observed from a frame of reference moving with the background velocity, U_o. Justification for this statement will be provided in a later section. The frequency Ω is defined by

$$\Omega \equiv \omega - K_x U_o \qquad (28)$$

The parameter G represents a convenient grouping. This grouping is defined by

$$G \equiv K_z - \frac{i\gamma g}{c^2} \qquad (29)$$

To obtain a non-trivial solution of system (27) the determinant of the four-by-four matrix must equal zero. Solving this determinant equation for K_z yields

$$K_z = i\,\frac{1}{2}\left(\frac{\gamma g}{c^2} + \frac{d\ell n\Omega}{dz}\right) \pm \left\{\frac{\left[\Omega^2 - (\phi + \omega_a^2 + \frac{gK_x}{\Omega}\frac{d\Omega}{dz})\right]}{c^2} + \frac{K_x^2\left[\left(\phi + \omega_g^2 - \frac{1}{4}\left(\frac{dU_o}{dz}\right)^2\right) - \Omega^2\right]}{\Omega^2}\right\}^{1/2} \qquad (30)$$

Several new parameters are introduced in equation (30). These are

$$\phi \equiv g\,\frac{d\ell nT_o}{dz} \qquad (31)$$

a frequency ω_a which is termed the acoustic cut-off frequency, where

$$\omega_a \equiv \frac{\gamma g}{2c} \tag{32}$$

and a frequency ω_g termed the Brunt-Väisälä frequency, where

$$\omega_g \equiv \frac{(\gamma-1)^{1/2} g}{c} \tag{33}$$

Equation (30) is seen to consist of an imaginary term and a term which is either real, imaginary, or zero. The physical significance of these terms, and of the frequencies ω_a and ω_g is more evident if equation (30) is specialized to the case in which there are no background winds or mean temperature gradients. In this case

$$K_z = i \frac{\omega_a}{c} \pm \left[\frac{(\omega^2 - \omega_a^2)}{c^2} + \frac{K_x^2(\omega_g^2 - \omega^2)}{\omega^2} \right]^{1/2} \tag{34}$$

Figure 2-1 is a graph obtained by equating the second term on the right in equation (34) to zero. The curves in the ω, K_x plane separate two regimes, within which K_z has both a real and an imaginary part, from a regime in which K_z is purely imaginary and only horizontal wave propagation is possible.

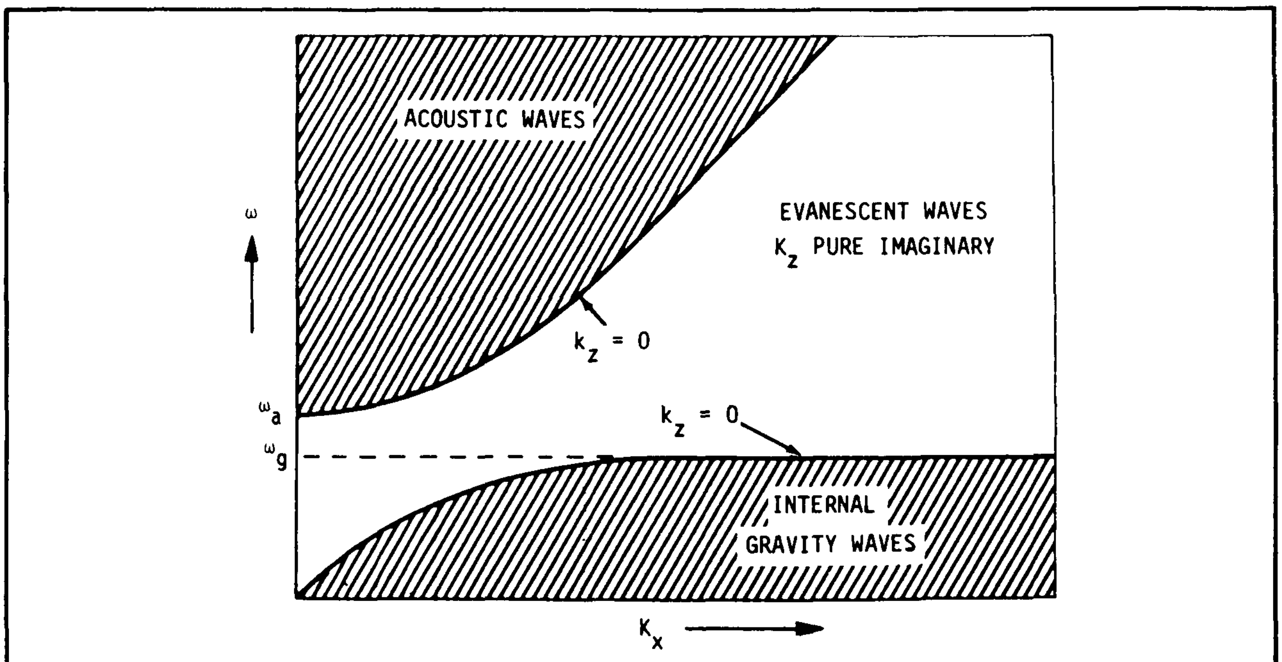


Figure 2-1. WAVE REGIMES IN THE ω - K_x PLANE FOR AN "ISOTHERMAL" ATMOSPHERE LACKING BACKGROUND WINDS

The $i\,\omega_a/c$ term in equation (34) has significance in terms of the wave kinetic energy, E_k, which may be defined by

$$E_k \equiv \frac{\rho_o}{2}\left[|u'|^2 + |w'|^2\right] \tag{35}$$

Equation (25) for the ground state, together with equations (26) for u' and w', then permits equation (35) to be reduced to the form

$$E_k = \frac{\rho_o(z_o)}{2}\,(X^2 + Z^2) \tag{36}$$

Thus, waves propagate with constant kinetic energy, but vary in amplitude, when the imaginary part of K_z equals $i\omega_a/c$ for the gravity or acoustic cases. This condition is only satisfied in a "windless" atmosphere having a constant mean temperature.

A dispersion relation for either gravity or acoustic waves that includes the effects of wind shear and temperature gradients is obtained by equating k_z to the real part of K_z in equation (30). The parameter k_z is, of course, a constant since K_z has been designated a complex constant in the formulation of the problem. The resulting dispersion relation is

$$\Omega^4 - \Omega^2\left[c^2(K_x^2 + k_z^2) + \omega_a^2 + \phi + \left(\frac{\gamma}{2} - 1\right)g\,\frac{d\ell n\Omega}{dz}\right] + k_z^2c^2\left[\omega_g^2 + \phi - \frac{1}{4}\left(\frac{dU_o}{dz}\right)^2\right] = 0 \tag{37}$$

Further, it is convenient to define an amplification or attenuation factor, ℓ_z, which is the imaginary part of K_z and is also a constant by precondition. That is,

$$\ell_z \equiv \frac{1}{2}\left(\frac{\gamma g}{c^2} + \frac{d\ell n\Omega}{dz}\right) \tag{38}$$

According to equation (30), Ω, ℓ_z, k_z and the ratios Q:P:X:Z must be constant for K_z to be a complex constant. This limits the range of parameters over which equation (26) provides a valid model of gravity wave processes. This limitation will be considered in more detail in subsection 3.2

From equation (26) variations in wind speed and density are seen to be related by

$$\frac{\rho'}{\rho_o} = \left(\frac{Q}{X}\right) u' \tag{39}$$

By using three of the equations in system (27) it is possible to solve for P/X, Z/X, and Q/X. These three equations may be represented by the matrix equation

$$\begin{pmatrix} \Omega & -\dfrac{c^2 G}{\gamma} & -i\ g \\ -\gamma\left[G + \dfrac{ig(\gamma-1)}{c^2}\right] & \Omega & 0 \\ -(G - i\ \dot{\phi}/g) & 0 & \Omega \end{pmatrix} \begin{pmatrix} \dfrac{Z}{X} \\ \dfrac{P}{X} \\ \dfrac{Q}{X} \end{pmatrix} = \begin{pmatrix} 0 \\ \gamma K_x \\ K_x \end{pmatrix} \tag{40}$$

For Q/X, the application of Cramer's rule then gives

$$\frac{Q}{X} = \frac{K_x}{\Omega} \left\{ \frac{\dfrac{\omega_g^2}{c^2 k_z^2}\left[\left(1 + \dfrac{\phi}{\omega_g^2}\right)\left(\dfrac{\gamma}{2} + \dfrac{ic^2 k_z}{g} - \dfrac{c^2}{2g}\dfrac{d\ell n\Omega}{dz}\right) - \dfrac{\Omega^2}{\omega_g^2}\right]}{1 + \dfrac{\omega_a^2}{c^2 k_z^2}\left(1 + \dfrac{\phi}{\omega_a^2} - \dfrac{\Omega^2}{\omega_a^2}\right) + \dfrac{i}{k_z}\dfrac{d\ell n\Omega}{dz} - \dfrac{1}{4k_z^2}\left(\dfrac{d\ell n\Omega}{dz}\right)^2} \right\} \tag{41}$$

Equations (26), (37), and (41) will be used to develop an expression relating density and velocity variations for low frequency gravity waves in the presence of a background wind shear.

Subsequently, the dispersion relation and equation (41) will be specialized by the neglect of terms having relatively small magnitudes. The result of this process will be an asymptotic form of equation (39), which will apply when the square of the frequency in the wind system, Ω^2, is small compared to ω_g^2.

Section III

AN EXPRESSION RELATING DENSITY AND VELOCITY VARIATIONS

At this point it is possible to combine equations (37), (39), and (41) to produce an expression relating density variations to variations in the horizontal component of the wind velocity. However, the resulting expression would apply for both gravity and acoustic waves. Thus it would be more general, and hence more complex, than is necessary.

By making careful approximations based upon the relative orders of magnitude of terms in equations (37) and (40), the form of the desired expression can be simplified. The orders of magnitude of the terms in these equations will be based here on the typical values for the time and space scales of a gravity wave given by Hines (1960). With these values the square of the horizontal wave number K_x^2, is on the order of 10^{-8} m^{-2} and the square of the vertical wave number, k_z^2, is on the order of 10^{-6} m^{-2}. The angular frequency, ω, is seen to be on the order of 10^{-3} sec^{-1}. Also, for a speed of sound squared, c^2, on the order of 10^5 m^2/sec^2, the squares of the acoustic cut-off frequency, ω_a^2, and the Brunt-Väisälä frequency, ω_g^2, must be on the order of 10^{-3} sec^{-2}.

It therefore is clear that

$$\phi \lesssim \omega_g^2 < \omega_a^2 << k_z^2 c^2 \quad (42)$$

In addition to this, it is assumed that frequencies measured from a frame of reference moving with the wind are low compared to the Brunt-Väisälä frequency. That is,

$$\Omega^2 << \omega_g^2 \quad (43)$$

This approximation is the "windy-atmosphere" equivalent to the inequality used by Hines (1960) in his treatment of a quiescent atmosphere. Inequality (43) reduces to the inequality of Hines, $\omega^2 << \omega_g^2$, in the absence of background winds.

A further approximation is based upon data of Rosenberg (1968) for altitudes between 80 km and 100 km. These data suggest that values for the background wind, U_o, are on the order of 10^2 m/sec and values for the background wind shear, dU_o/dz, are on the order of 10^{-2} sec^{-1}. For these orders of magnitude it follows that

$$\left(\frac{\gamma}{2} - 1\right) g \frac{d\ln\Omega}{dz} << k_z^2 c^2 \tag{44}$$

3.1 AN ASYMPTOTIC LIMIT

Neglect of relatively small terms in the dispersion relation, equation (38), gives the expression

$$\frac{\Omega k_z}{K_x} = \pm \left[\omega_g^2 + \phi - \frac{1}{4}\left(\frac{dU_o}{dz}\right)^2\right]^{1/2} \tag{45}$$

The plus or minus sign on the right in this equation indicates that $\Omega k_z/K_x$ may take on either positive or negative values. When U_o equals zero, these signs simply indicate that the wave may propagate either upward or downward for any given horizontal direction of propagation. (k_z may be positive or negative relative to K_x.) For the more general case, described by equation (45), wind is significant. Here, the parameter Ω may also be positive or negative depending on whether or not the horizontal phase velocity, ω/K_x, exceeds the wind speed, U_o, in the horizontal direction of wave propagation.

It is convenient for subsequent discussion to define a Richardson's number, Ri, based on equation (45). Thus,

$$Ri \equiv \frac{(\omega_g^2 + \phi)}{\left(\frac{dU_o}{dz}\right)^2} \tag{46}$$

When Ri < 1/4, the grouping $\Omega k_z/K_x$ must be imaginary. There are two possible ways that this can occur. One possibility is that k_z is imaginary and Ω is real. In this case equation (45) describes evanescent waves which characteristically do not propagate in a vertical direction. (This conclusion may be justified further, for $\Omega^2 << \omega_g^2$, by reviewing equation (30) and the discussion following that equation.) The alternative, for an imaginary value of $\Omega k_z/K_x$, is that k_z be real and Ω be imaginary. In this case, the amplitude of the perturbation terms (equation (26)) vary in an exponential manner with time. The fluid medium is thus said to be unstable under perturbation. It is also possible for both Ω and k_z to be imaginary while the Richardson number satisfies the inequality, Ri > 1/4.

The "critical layers" of Bretherton (1966) and Booker and Bretherton (1967) occur when Ri = 1/4. In this case the horizontal wind, U_o, and the horizontal component of the phase velocity, ω/K_x, are in the same direction and are equal. It follows from equation (45) that Ri = 1/4 corresponds to zero in the frequency, Ω, which is observed from a coordinate system moving with the wind. The frequency Ω is defined by equation (28) in such a manner that it equals zero when $\omega/K_x = U_o$. It has been noted by Booker and Bretherton (1967) and Hines (1970) that these critical layers correspond to regions of the atmosphere within which momentum is preferentially transferred between the internal gravity waves and the background wind.

The neglect of relatively small terms in equation (41) gives an expression for Q/X which is valid in the same asymptotic limit as equation (45). Combining this asymptotic form for equation (41) with equation (45) gives

$$\frac{Q}{X} = \frac{(\omega_g^2 + \phi)^{1/2}}{g} \exp\left\{ i \left[\frac{\pi}{2} \pm \arctan(4Ri - 1)^{-1/2} \right] \right\}. \tag{47}$$

The relation between density and velocity variations in an atmosphere having wind shear and temperature gradients is determined by combining equations (39) and (47). Thus,

$$\frac{\rho'}{\rho_o} = u' \left[\frac{(\gamma-1)}{c^2} + \frac{1}{g} \frac{d\ell nT_o}{dz} \right]^{1/2} \exp\, i \left[\frac{\pi}{2} \pm \arctan\,(4Ri - 1)^{-1/2} \right] \tag{48}$$

Equation (48) is the principal result of this report. As the wind shear and temperature gradient approach zero, the relation obtained by Hines (1960) is approached. That is,

$$\frac{\rho'}{\rho_o} = \pm\, i\, \frac{(\gamma-1)^{1/2}}{c}\, u' \tag{49}$$

At critical layers, where Ri = 1/4, equation (48) reduces to

$$\frac{\rho'}{\rho_o} = \pm \frac{(\gamma-1)^{1/2}}{c}\, u' \tag{50}$$

3.2 BACKGROUND WIND SPEED AND MEAN TEMPERATURE

According to equations (27) and (30), the parameters Ω, ℓ_z and k_z and the ratios Q:P:Z:X must be constant in order for K_z to be a complex constant. Thus, equation (26) represents a valid gravity wave model only if one considers those mean temperature and and background wind speed profiles which preserve the z-independence of ℓ_z, k_z and of the ratios Q:P:Z:X. This subsection is concerned with demonstrating the conditions under which a set of linearly altitude-dependent profiles can preserve the z-independence of ℓ_z, k_z and of the ratios Q:P:Z:X.

In summary it may be noted that

$$\Omega \equiv \omega - K_x U_o \tag{51}$$

$$\ell_z \equiv \frac{1}{2}\left[\frac{\gamma g}{c^2} + \frac{\partial \ln \Omega}{\partial z}\right] \tag{52}$$

and

$$k_z \equiv \pm\left\{\frac{\left[\Omega^2 - \left(\phi + \omega_a^2 + gK_x \frac{d\ln\Omega}{dz}\right)\right]}{c^2} + \frac{K_x\left[\left(\phi + \omega_g^2 - \frac{1}{4}\left(\frac{dU_o}{dz}\right)^2\right) - \Omega^2\right]}{\Omega^2}\right\}^{1/2} \tag{53}$$

where ω and K_x are assumed to be constant.

Also, the application of Cramer's rule to system (40) provides expressions for Δ, $P\Delta/X$, $Q\Delta/X$, and $Z\Delta/X$ which may be used to demonstrate the z-independence of the ratios Q:P:Z:X. These are:

$$\Delta \equiv \Omega^2 - \Omega[c^2(k_z + i\ell_z)(k_z + i\ell_z - \frac{i\gamma g}{c^2}) + \phi] \tag{54}$$

$$Z\Delta/X = \Omega K_x c^2 \left[k_z + i\ell_z - \frac{i(\gamma-1)g}{c^2}\right] \tag{55}$$

$$P\Delta/X = \gamma K_x[\Omega^2 - (\gamma-1)g^2/c^2 - \phi] \tag{56}$$

$$Q\Delta/X = K_x[\Omega^2 - \frac{ic^2}{g}(k_z + i\ell_z - \frac{i\gamma g}{c^2})(\phi + \frac{g^2(\gamma-1)}{c^2})] \tag{57}$$

Inspection or substitution will demonstrate that the quantities on the left in equations (51) through (57) are constant for the linear profiles.

$$U_o(z) = U_o(z_o) + \left(\frac{dU_o}{dz}\right)_{z_o}(z - z_o) \tag{58}$$

$$T_o(z) = T_o(z_o) + \left(\frac{dT_o}{dz}\right)_{z_o}(z - z_o) \tag{59}$$

provided that both of the auxiliary conditions

$$\left|\left(\frac{d\ell nU_o}{dz}\right)_{z_o}(z - z_o)\right| \lesssim 0.1 \text{ and } \left|\left(\frac{d\ell nT_o}{dz}\right)_{z_o}(z - z_o)\right| \lesssim 0.1 \tag{60}$$

are satisfied. Conditions (60) are therefore the desired conditions under which a set of linearly altitude-dependent profiles (equations (58) and (59)) preserve the z-independence of ℓ_z, k_z and of the ratios Q:P:Z:X. Equations (26) represent a valid gravity wave model for the background profiles (58) and (59) provided that conditions (60) are satisfied.

Under conditions (60), equations (58) and (59) may be used together with equation (48) and wind data to estimate density variations. Equations (58) and (59) may be of value in propagation analyses where $(z - z_o)$ may be considered to be the thickness of an atmospheric layer. In this case, conditions (60) may be used to specify the thickness of a layer. Both of these conditions must be satisfied on a layer-by-layer basis for a given set of ground state profiles. In another sense, conditions (60) may be used to define a region within which a particular density-variation estimate is valid. This is possible for background wind and temperature profiles which are approximated in the vicinity of z_o by a linear altitude dependence.

3.3 ATTENUATION OR AMPLIFICATION OF GRAVITY WAVES

When the real and imaginary parts of K_z are substituted into equation (26) the resulting expression is of the form

$$\frac{\rho'}{\rho_o Q} = \frac{P'}{P_o P} = \frac{u'}{X} = \frac{w'}{Z} = e^{\ell_z z} e^{i(\omega t - K_x x - k_z z)} \tag{61}$$

The constant ℓ_z therefore describes the amplification or attenuation of the perturbation quantities which appear in equation (61).

Expressing equation (38) in terms of the ground state profiles, $T_o = T_o(z)$ and $U_o = U_o(z)$, which are discussed in the previous subsection, gives

$$\ell_z = \frac{1}{2}\left[\frac{g}{R\ T_o(z)} + \frac{dU_o(z)}{dz}\left(U_o(z) - \frac{\omega}{K_x}\right)^{-1}\right] \tag{62}$$

The magnitude and sign of ℓ_z at the point z_o may be established from equation (62). Criteria for specifying the neighborhood of z_o within which ℓ_z is practically constant are given in the previous subsection.

At a critical point ℓ_z must be infinite in magnitude and must depend in sign on the sign of dU_o/dz. As mentioned in subsection 3.1 these critical layers have been found to correspond to regions of the atmosphere within which momentum is preferentially transferred between internal gravity waves and the background wind. These layers are therefore regions within which the assumptions of this perturbation analysis break down.

Section IV

THE PHYSICAL INTERPRETATION OF RESULTS

This section is intended to justify, and perhaps clarify, the physical significance attached to certain parameters in this analysis. Interrelations between parameters are also discussed. A Galilean transformation relating a coordinate system moving with wind speed, U_o, to a rest or laboratory system is used to clarify the relationship between Ω and ω. Information is also obtained from graphs in the $\Omega/\omega - \omega/K_x U_o$ plane, which are based on the asymptotic dispersion relation and the definition of Ω. In addition to this, the results of Section III are used in making an estimate of the magnitude of density variations associated with particular sets of wind data.

4.1 THE PHYSICAL SIGNIFICANCE OF Ω

In previous discussion, the parameter Ω has been interpreted as the "frequency in a frame of reference moving with the wind speed U_o". This statement can be justified by a simple Galilean transformation.

The space and time periodicity of gravity waves is described by the factor

$$e^{i(\omega t - K_x x - k_z z)} \tag{63}$$

Suppose another coordinate system is defined which moves with the mean wind velocity $U_o(z)$, for any given z. The Galilean transformation that relates the fixed and moving systems is, therefore,

$$\left.\begin{aligned} x &= x' + U_o t' \\ z &= z' \\ t &= t' \end{aligned}\right\} \tag{64}$$

where the primed coordinates locate positions and times with respect to the moving system. Substitution into equation (63) from equation (28) followed by use of equations (64) shows that

$$e^{i(\omega t - K_x x - k_z z)} = e^{i(\Omega t' - K_x x' - k_z z')} \tag{65}$$

Thus, the amplitude of the periodicity remains unchanged under transformation and only the frequencies change. It is clear from equation (65) and transformation (64) that Ω is the frequency of a gravity wave in a coordinate system moving with the background wind velocity U_o.

4.2 A GRAPHICAL METHOD OF SHOWING PARAMETRIC RELATIONSHIPS

This subsection is devoted to developing a method for showing the constraints imposed by the asymptotic dispersion relation upon the parameters in equation (28). These parametric constraints are best shown in the $\Omega/\omega - \omega/K_xU_o$ plane.

Equation (28), which defines Ω, may be written in the form

$$\frac{\Omega}{\omega} = 1 - \left(\frac{\omega}{K_xU_o}\right)^{-1} \tag{66}$$

which is conveniently plotted in the $\Omega/\omega - \omega/K_xU_o$ plane.

Furthermore, the dispersion relation for low frequency gravity waves in an atmosphere with a constant mean temperature is given by

$$\left(\frac{\Omega k_z}{K_x\omega_g}\right)^2 = \left[1 - \frac{1}{4Ri}\right] \tag{67}$$

where, for this "isothermal" case

$$Ri \equiv \frac{\omega_g^2}{\left(\frac{\partial U_o}{\partial z}\right)^2} \tag{68}$$

Since evanescent waves are of primary concern here, it will be presumed that $k_z^2 > 0$. Under this condition it is evident, from the fact that Ri is non-negative, that

$$0 \leq \left(\frac{\Omega k_z}{K_x U_o}\right)^2 \leq 1 \tag{69}$$

Conditions (69), which are derived from the dispersion relation (equation (67)) for constant mean temperature, define two limiting cases. Furthermore, it is possible for conditions (69) to be written in forms which allow them to be conveniently interpreted in terms of the Ω/ω - $\omega/K_x U_o$ plane. These forms are

$$\frac{\Omega^2}{\omega^2} \leq \left(\frac{\omega_g}{k_z U_o}\right)^2 \left(\frac{\omega}{K_x U_o}\right)^{-2} \tag{70}$$

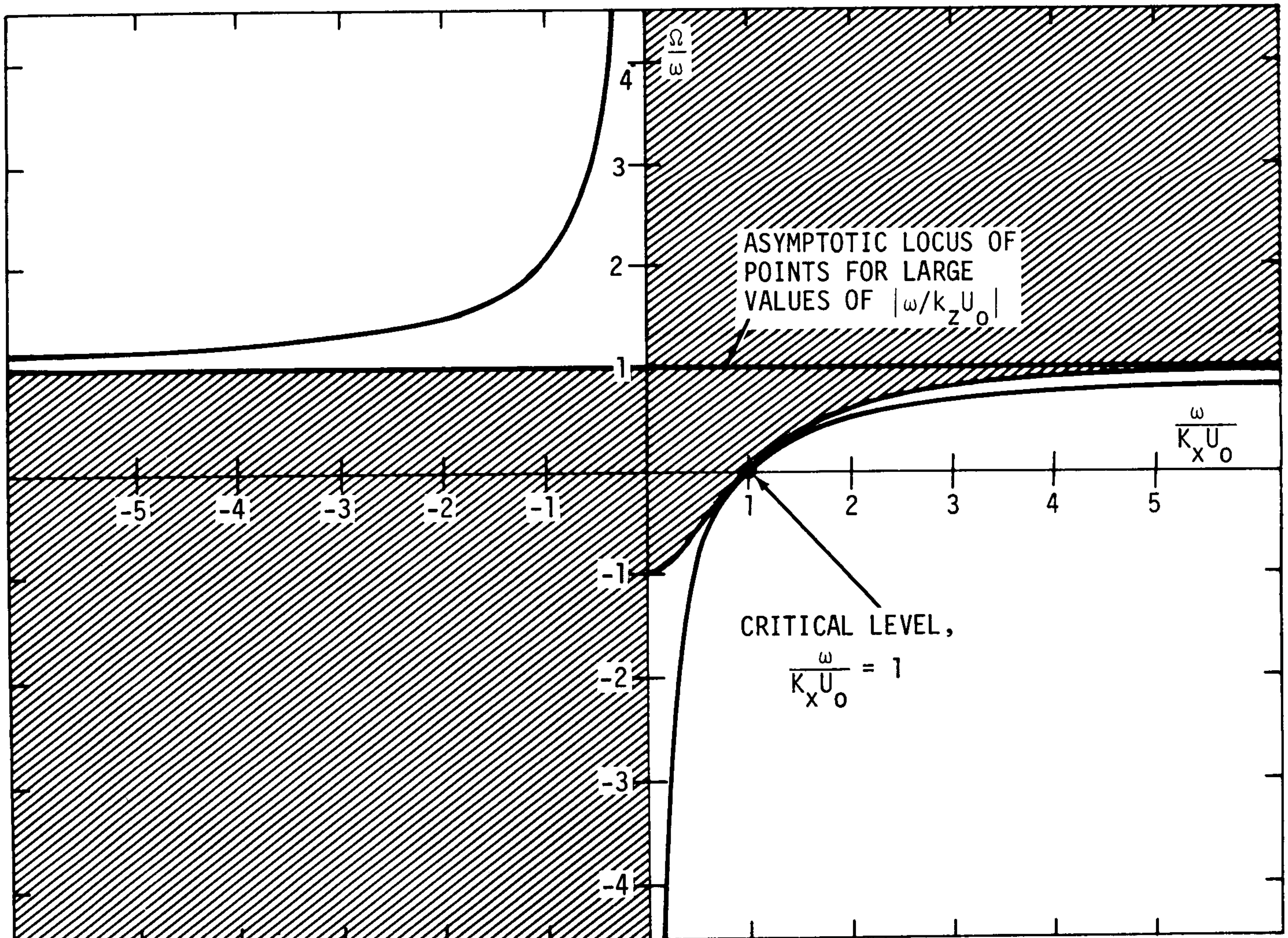
and

$$\left(1 - \frac{\Omega}{\omega}\right)\left[\frac{\Omega}{\omega} - 1 + 2\left(\frac{\omega^2}{K_x^2 U_o^2} + 1\right)^{-1}\right] \leq 0 \tag{71}$$

Reference to equations (67) and (68) will verify that condition (70) defines the portions of the Ω/ω - $\omega/K_x U_o$ plane for which the wind-shear, $\partial U_o/\partial z$, is non-zero. In addition, consideration of these same equations verifies that, when k_z^2 is greater than zero, equation (71) defines portions of the Ω/ω - $\omega/K_x U_o$ plane for which low-frequency gravity waves are stable.

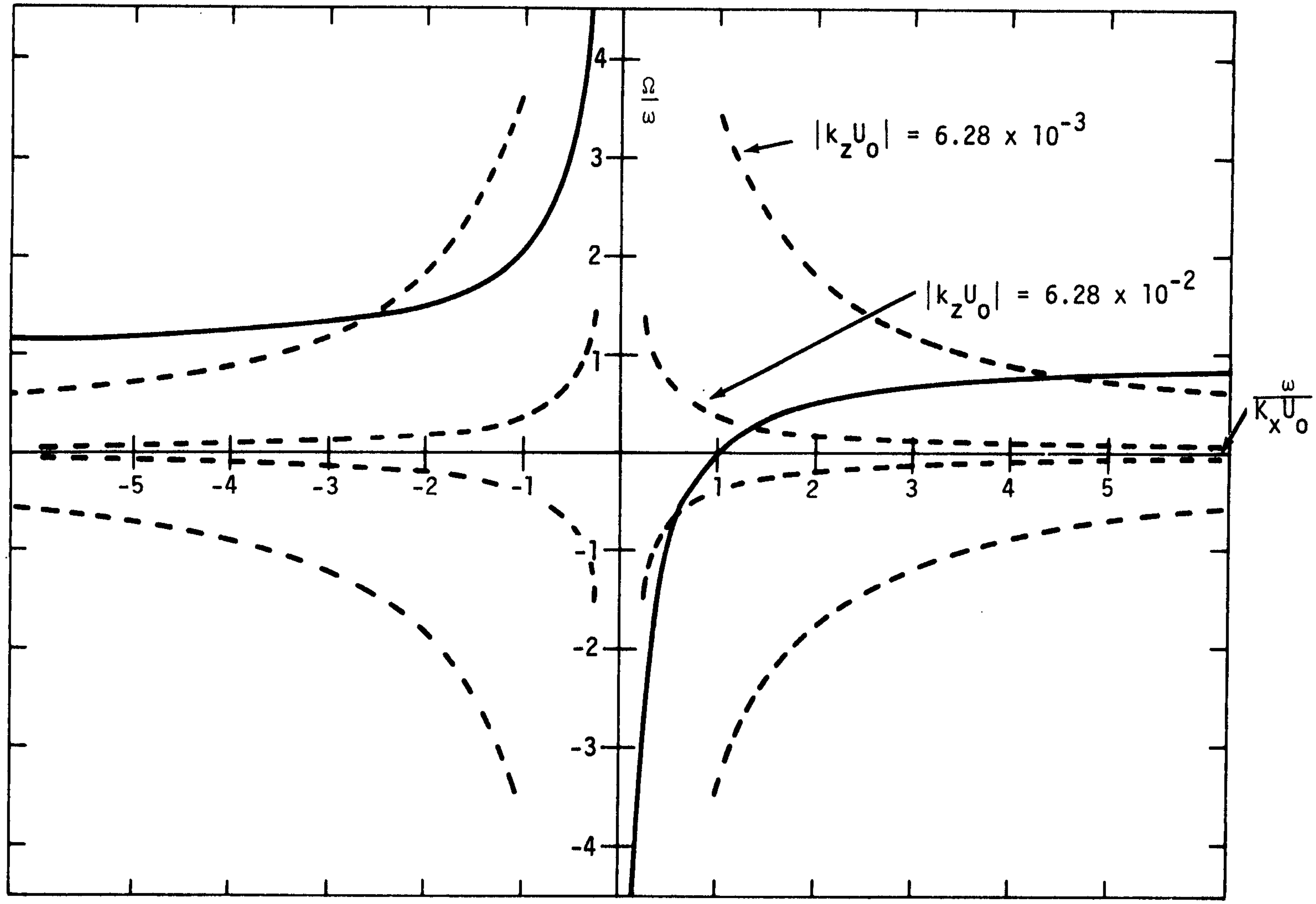
Figures 4-1a and 4-1b are simply a graphical representation of the constraints imposed upon equation (66) by conditions (70) and (71). Thus, stability and non-zero shear conditions derived from the dispersion equation are used to qualify equation (66). Equation (66) is nothing more than the parametric relationship associated with the Doppler effect.

The equality in equation (71) defines the boundary of the stable regime. Similarly, the equality in equation (70) results in four hyperbolas in the Ω/ω - $\omega/K_x U_o$ plane which have the Ω/ω and $\omega/K_x U_o$ axes as asymptotes. A different set of hyperbolas is found for each value of $(\omega_g/k_z U_o)^2$. The region between a particular set of hyperbolas and the axes corresponds to the region within which wind shear, $\partial U_o/\partial z$, must be non-zero. The hyperbolas thus form the boundary of a $(\omega_g/k_z U_o)^2$ dependent regime.



NOTE: THE SHADED PORTIONS CORRESPOND TO REGIONS IN WHICH GRAVITY WAVES ARE NOT ALLOWED. THE UNSHADED PORTIONS CORRESPOND TO STABLE GRAVITY WAVE REGIMES. THE SOLID HYPERBOLAS ARE THE LOCUS OF ALL POSSIBLE POINTS.

Figure 4-1a. PARAMETRIC REGIMES FOR GRAVITY WAVES IN AN "ISOTHERMAL" ATMOSPHERE



NOTE: THE DASHED HYPERBOLAS ARE CALCULATED FOR $\omega_g^2 = 5.3 \times 10^{-4}$ SEC^{-2}.

THE SOLID HYPERBOLAS CORRESPOND TO THE LOCUS OF ALL POSSIBLE POINTS, WHILE THE REGION BETWEEN THE DASHED HYPERBOLAS AND THE AXES INCLUDES POINTS FOR WHICH NON-ZERO WIND SHEARS MAY BE TREATED.

Figure 4-1b. PARAMETRIC REGIMES FOR GRAVITY WAVES IN AN "ISOTHERMAL" ATMOSPHERE

The curves, in the $\Omega/\omega - \omega/K_xU_o$ plane, associated with equation (66) are two hyperbolas having the Ω/ω axis and the line, $\Omega/\omega = 1$, as asymptotes. The hyperbolas lie in the portions of the $\Omega/\omega - \omega/K_xU_o$ plane corresponding to $\Omega/\omega \geq 1$, $\omega/K_xU_o \leq 0$, and to $\Omega/\omega \leq 1$, $\omega/K_xU_o \geq 0$.

Two points on the curves defined by equation (66) should be noted. In the absence of wind, points in the $\Omega/\omega - \omega/K_xU_o$ plane collapse to the point at ($\Omega/\omega = 1$, $\omega/K_xU_o = \infty$). Another important point corresponds to the critical layer and has the coordinates, ($\Omega/\omega = 0$, $\omega/K_xU_o = 1$). It is interesting that this latter point is "just stable". That is, it satisfies the equality in the stability condition, $\Omega/\omega \leq 1 - 2\left(\frac{\omega^2}{K_x^2U_o^2} + 1\right)^{-1}$. At no other point in these figures does the locus of points given by equation (66) appear to approach the boundary of the stability regime.

To improve comprehension of Figures 4-1a and 4-1b, the physical meaning associated with different portions of the $\Omega/\omega - \omega/K_xU_o$ plane can be considered. The portion of this plane for which $\omega/K_xU_o > 0$ corresponds to physical situations in which the horizontal component of the phase velocity and the wind are in the same direction. In contrast, the half-plane for which $\omega/K_xU_o < 0$ corresponds to cases in which the wind opposes the horizontal component of the phase velocity.

Similarly, the half of the $\Omega/\omega - \omega/K_xU_o$ plane for which $\Omega/\omega > 0$ corresponds to physical situations in which time sequences, which are measured both in the wind coordinate system and in the rest coordinate system, exhibit the same ordering. In contrast to this, points in the half plane for which $\Omega/\omega < 0$ have in common a physical situation in which the time-sequences measured from the two coordinate systems are opposite in order.

The latter case, in which the time sequences are oppositely ordered, only occurs when the wind is in the same direction as the horizontal component of the phase velocity. In fact, it is restricted to values of ω/K_xU_o which lie between unity and zero. A simple sketch illustrates that, when the wind speed in the direction of horizontal propagation exceeds the phase velocity in that direction, the time-sequence in the rest system will become exactly the reverse of the time-sequence of measured events for the wind system. When the specified wind speed is less than the horizontal phase velocity component, the same time-ordering of events is measured in both systems.

Nominal values for the horizontal component of the phase velocity, from Hines (1960), suggest that 10m/sec < ω/K_x < 10^2m/sec at altitudes between 80 km and 100 km. Data of Rosenberg (1968) for the 80 km to 100 km region suggest that mean wind speeds on the order of 10^2m/sec are not unusual. Thus, both positive and negative values for Ω/ω may indeed occur for the 60 km to 100 km altitude range.

When the wind opposes the horizontal phase-velocity component ($\omega/K_xU_o < 0$), the frequency measured in the rest system is always less than that measured in the wind system. These two frequencies become equal as ω/K_xU_o approaches a large negative value.

An interesting conclusion results from the trends seen in Figure 4-1b. This figure shows that when U_o is in the direction of horizontal wave propagation, only the region in the vicinity of a critical point may simultaneously exhibit a high wind speed, U_o; wind shear, $\partial U_o/\partial z$; and a short vertical wavelength, λ_z. This suggests that "jet-like" motion in the atmosphere only corresponds to a critical layer for gravity waves with the horizontal component of phase velocity in the same direction as the jet and with relatively short vertical wavelengths.

4.3 THE MAGNITUDE OF DENSITY VARIATIONS AT METEOR HEIGHTS

This subsection presents examples of how equation (48) may be used to estimate magnitudes for ρ'/ρ_o from wind data. Equation (48) shows the magnitude of ρ'/ρ_o to be given by

$$\left(\frac{\rho'}{\rho_o}\right)_m = \left[\frac{\gamma-1}{c^2} + \frac{1}{g}\frac{d\ell nT_o}{dz}\right]^{1/2} (u')_m \tag{72}$$

As discussed in the Appendix, equation (72) probably provides the best estimate of gravity wave density variations when applied to an ensemble average such as that of Rosenberg (1968). By statistically analyzing 70 midlatitude wind profiles between 90 and 150 km, Rosenberg shows that r.m.s. wind speeds range from 45 m/sec to 70 m/sec. At 90 km the U. S. Standard Atmosphere (1962) gives a value for the mean temperature of 180.65°K. The temperature gradient at this altitude is 0.75×10^{-3} °K/m so that the temperature gradient term in equation (72) is on the order of plus 0.42×10^{-6} $(\mathrm{sec/m})^2$. The speed of sound is about 270 m/sec so that for these values equation (72) becomes,

$$\left.\frac{\rho'}{\rho_o}\right|_m = (2.43 \times 10^{-3})(u')_m$$

Using the r.m.s. values for $(u')_m$, from Rosenberg (1968), shows that

$$\left.\frac{\rho'}{\rho_o}\right|_m \sim 0.11 - 0.17$$

Thus $|\rho'|$ is about 10 to 20 percent of ρ_o at around 90 km.

Section V
SUMMARY

This report gives a brief review of gravity wave theory. A method is also developed for determining the density variations from observed wind speed profiles between 60 km and 100 km. The principal result is that the magnitude of density variation is independent of the background wind speed and background wind shear under the approximations outlined in Section III. However, this magnitude may depend upon the ratio of the mean temperature, T_o, to the mean temperature gradient, $\partial T_o/\partial z$.

An analysis of the averaging techniques (see the Appendix) suggests that unacceptably large altitude ranges are involved in determining the mean temperature or background wind speed from individual profiles of the form $\xi = \xi(x_o, z, t_o)$. Ensemble averaging thus appears to be the best method of estimating these background or mean values. A representative set of wind speed values have been obtained by Rosenberg (1968) by the statistical analysis of 70 midlatitude wind profiles between 90 km and 150 km suggest that gravity wave density variations may be on the order of 10 to 20 percent of the local mean density.

Future work should be directed toward processing wind profiles using the methods developed here. Consideration should also be given to the effects of reflection and ducting upon the perturbations induced by gravity waves.

Section VI

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Appendix

ESTIMATION OF BACKGROUND WIND AND MEAN TEMPERATURE FROM DATA

For convenience, general wind and temperature profiles will be represented in this appendix by a common parameter ξ. The following arguments are valid for either type of data. Thus,

$$\left.\begin{matrix} u(x, z, t) \\ T(x, z, t) \end{matrix}\right\} \equiv \xi(x, z, t) \tag{A-1}$$

The theory developed in this report includes certain assumptions concerning the form of $\xi(x, z, t)$. That is,

$$\xi(x, z, t) \equiv \xi_o(z) + \xi'(x_o, z_o, t_o)\, e^{i[\omega t - K_x x - (k_z + i\ell_z)z]} \tag{A-2}$$

where $\xi_o(z)$ is the mean or background value.

Wind and temperature data obtained by any of the conventional means listed in Section I have a common characteristic. These data are available for a relatively large range of z values in comparison to a much more limited x and t ranges. Parametrically this can be stated by requiring that

$$\xi \simeq \xi(x_o, z, t_o) \tag{A-3}$$

for these data.

Given data of the sort characterized by equation (A-3), it is tempting to try to define $\xi_o(z_o)$ by some sort of average over values of $\xi(x_o, z, t_o)$ centered on z_o. Criteria are thus developed in this appendix under which

$$\left\langle \xi(x_o, z, t_o) \right\rangle_z \equiv \frac{1}{2\Delta z} \int_{z_o - \Delta z}^{z_o + \Delta z} \xi(x_o, z, t_o)\,dz \approxeq \xi_o(z_o) \tag{A-4}$$

It is assumed here that the conditions presented in subsection 3.2 are valid. If so then background or mean profiles of the form

$$\xi_o(z) = \xi_o(z_o) + \left(\frac{d\xi_o}{dz}\right)_{z_o} (z - z_o) \tag{A-5}$$

are consistent with the gravity wave model presented in this report.

Substitution from equations (A-2) and (A-5) into equation (A-4) followed by integration gives

$$\left\langle \xi(x_o,\ z,\ t_o) \right\rangle_z = \xi_o(z_o) + \frac{\xi'(x_o,\ z_o,\ t_o)}{(k_z^2 + \ell_z^2)\ z} (a + ib) \tag{A-6}$$

where

$$\begin{aligned} a &\equiv [k_z \cosh(\ell_z \Delta z)\sin(k_z \Delta z) + \ell_z \sinh(\ell_z \Delta z)\cos(k_z \Delta z)] \\ b &\equiv -[k_z \sinh(\ell_z \Delta z)\cos(k_z \Delta z) - \ell_z \cosh(\ell_z \Delta z)\sin(k_z \Delta z)] \end{aligned} \tag{A-7}$$

Since

$$a + ib \equiv [a^2 + b^2]^{1/2}\ e^{i\phi} \tag{A-8}$$

where the phase angle, ϕ, is given by

$$\phi \equiv \arctan b/a \tag{A-9}$$

equation (A-6) may be written in the form

$$\left\langle \xi(x_o,\ z,\ t_o) \right\rangle_z = \xi_o(z_o) + \frac{\xi'(x_o,\ z_o,\ t_o)}{(k_z^2 + \ell_z^2)^{1/2}\ \Delta z} \left[\sin^2(k_z \Delta z) + \sinh^2(\ell_z \Delta z)\right]^{1/2} e^{i\phi} \tag{A-10}$$

If absolute values are considered, it follows immediately from equation (A-10) that

$$\left\langle \xi(x_o,\ z,\ t_o) \right\rangle_z \simeq \xi_o(z_o) \tag{A-11}$$

provided that

$$\left| \frac{\xi'(x_o,\ z_o,\ t_o)[\sin^2(k_z \Delta z) + \sinh^2(\ell_z \Delta z)]^{1/2}}{\xi_o(z_o)(k_z^2 + \ell_z^2)^{1/2}\ \Delta z} \right| \lesssim\ 0.1 \tag{A-12}$$

Condition (A-12) will now be investigated in order to determine what sort of Δz is necessary so that equation (A-11) is a valid approximation.

Consider wind data for an isothermal atmosphere, free of wind shear, which has a scale height, H. For this case, $\ell_z \sim 1/2H$ and $k_z \sim 2\pi/H$. Thus, for $u(x_o,\ z,\ t_o)$, condition (A-12) becomes

$$\left| \frac{u'(x_o,\ z_o,\ t_o)[\sin^2(2\pi \frac{\Delta z}{H}) + \sinh^2(\frac{\Delta z}{2H})]^{1/2}\ H}{5\ U_o(z_o)\Delta z} \right| \lesssim 0.1 \tag{A-13}$$

If, as a worst case, u' is presumed to be on the order of U_o one sees that the absolute value of the left hand term in (A-13) is on the order of 0.1 provided that Δz is on the order of 2H. Since $\lambda_z \sim H$, this implies that a suitable mean value of $U_o(z_o)$ can only be obtained by averaging $u(x_o,\ z,\ t_o)$ over values out to $2\lambda_z$ on either side of z_o. An averaging interval of this size is unacceptable. If $u'(x_o,\ z_o,\ t_o)$ is known to be much less than $U_o(z_o)$ the averaging interval may possibly be reduced to a more acceptable size.

Since atmospheric wind data may not generally exhibit the feature that $u'(x_o,\ z_o,\ t_o) \ll U_o(z_o)$, it would appear from the results of this appendix that the approximation expressed by (A-11) is not a useful means of determining $U_o(z_o)$ because of the unacceptably large altitude ranges which are required to validate the approximation. Perhaps a more valid approach is that of Rosenberg (1968) in which an ensemble average of $u(x_o, z, t_o)$ profiles is used to determine $U_o(z_o)$. The excursions about this mean may then be presumed to approximate the magnitude of $u'(x_o, z_o, t_o)$. In essence, this involves replacing a conventional average over a given altitude range by an ergodic hypothesis.

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